Home biased expectations and macroeconomic imbalances in a monetary union*

Dennis Bonam\textsuperscript{1} and Gavin Goy\textsuperscript{2}

\textsuperscript{1}De Nederlandsche Bank, \texttt{d.a.r.bonam@dnb.nl}
\textsuperscript{2}University of Amsterdam, \texttt{g.w.goy@uva.nl}

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Abstract

Under monetary union, economic dynamics may diverge across countries due to regional inflation differentials and a pro-cyclical real interest rate channel, yet stability is generally ensured through endogenous adjustment of the real exchange rate. The speed of adjustment depends, inter alia, on the way agents form expectations. We propose a model in which agents’ expectations are largely based on domestic variables, and less so on foreign variables. We show that such home bias in expectations strengthens the real interest rate channel and causes country-specific shocks to generate larger and more prolonged macroeconomic imbalances.

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1 Introduction

Ever since its formation, the euro area bore large and persistent external trade imbalances across its member states. While ‘core countries’, such as Germany, the Netherlands and Finland, have been running large trade surpluses, ‘periphery countries’, such as Greece, Portugal and Spain, faced consecutive trade deficits. Although one might expect such macroeconomic imbalances to arise naturally under monetary union, e.g. through some process of catching up, it is not entirely clear what underlies the persistence of these imbalances. In this paper, we argue that a departure from rational expectations and cross-country heterogeneity in expectations potentially obstruct macroeconomic adjustment in a monetary union.\(^1\)

Figure 1 offers some suggestive evidence in favor of this argument. It shows the regional dispersion in 1-year ahead inflation expectations and trade balance ratios, both measured by the cross-country standard deviation, across a group of euro area countries. In the left panel, inflation expectations are based on qualitative responses to the European Commission Business and Consumer Survey. Since quantification of these responses is based on a set of assumptions (laid out in the Appendix) we also consider inflation expectations from Consensus Forecasts in the right panel as a robustness. The figure suggests that greater regional dispersion in both measures of inflation expectations are associated with greater trade imbalances within the euro area.\(^2\) The aim of this paper is to develop a model that can explain both the observed regional dispersion in expectations and its positive relationship with macroeconomic imbalances in a monetary union.

Our starting point is a standard New Keynesian model for a two-country monetary union along the lines of Benigno (2004). Expectations, however, are backward-looking and updated

\(^1\)Of course, there may be other factors that hinder the reversal of macroeconomic imbalances, such as limited labor mobility across member states, imperfect financial integration and nominal rigidities (see e.g. Blanchard, 2007). Here, we focus only on regional dispersion in expectations as a possible novel source of the persistence in macroeconomic imbalances in a monetary union.

\(^2\)A similar observation is reported by Hoffmann et al. (2017), who show that US current account dynamics are strongly correlated with changes in survey expectations on long-run growth for the US relative to growth expectations for the rest of the world.
Figure 1: Regional dispersion in inflation expectations and trade balance ratios in the euro area, 1997Q1-2007Q4

Notes: The figure shows the standard deviation of 1-year ahead inflation expectations (horizontal axis) and the trade balance ratio (vertical axis) across Belgium, Germany, Ireland, Spain, France, Italy, the Netherlands, Austria, Portugal and Finland over the period 1997Q1-2007Q4. Inflation expectations were extracted from the European Commission Business and Consumer Survey (see the Appendix for more details) and Consensus Forecasts. The trade balance ratio measures the difference between exports and imports of goods and services as a share of GDP. Source: European Commission Business and Consumer Survey, Consensus Forecasts, OECD Quarterly National Accounts and own calculations.
periodically using a recursive least squares algorithm, as in Evans and Honkapohja (2001). Our main contribution is that we allow expectations to be home biased. In particular, we assume that agents, while acquiring information, focus more strongly on domestic variables than on foreign variables. Since the home bias in expectations may arise for many different reasons, we assume it to be exogenous and model it parsimoniously as a parameter that weighs down foreign variables in agents’ information set. It is because of this home bias that expectations may differ across the two countries in our model. We show that the stronger is the degree of home bias in expectations, the more amplified are country-specific disturbances and the larger and more prolonged are macroeconomic imbalances within the monetary union.

This result arises from a tug-of-war between a pro-cyclical real interest rate channel and a counter-cyclical real exchange rate channel that jointly determine the size and duration of macroeconomic imbalances in a monetary union. For instance, when a country experiences a positive, country-specific demand shock, inflation rises causing the domestic ex-ante real interest rate to be relatively low. Through the real interest rate channel, consumption then rises which pushes up aggregate demand and inflation, and lowers the real interest rate further. This pro-cyclical process leads to a deepening of the external trade deficit. At the same time, higher domestic prices cause the real exchange rate to appreciate and prompt a shift of consumption towards foreign goods. Through the real exchange rate channel, domestic firms lower their prices to regain competitiveness, which lowers inflation, raises the real interest rate and improves the trade balance. This process continues until the real interest rate and the trade balance are returned to steady state.

The relative strengths of the two channels depends, inter alia, on how expectations are formed. The backward-looking nature of expectations assumed in our model reinforces the feedback between inflation and inflation expectations, thereby strengthening the real interest

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3There is broad empirical evidence that supports the notion of sluggish adjustment of expectations, in particular inflation expectations (e.g. Carroll, 2003, Orphanides and Williams, 2005b, Orphanides and Williams, 2005a, and Coibion and Gorodnichenko, 2012). Milani (2007) and Slobodyan and Wouters (2012) show that allowing for adaptive expectations also increases a model’s fit to the data.
rate channel. Moreover, as expectations are home biased, agents do not (fully) incorporate the relative change in domestic versus foreign prices and thereby underestimate changes in the real exchange rate. Therefore, the home bias in expectations weakens the real exchange rate channel and slows down the process of macroeconomic realignment. As a consequence, country-specific disturbances generate larger and more prolonged macroeconomic imbalances if expectations are home biased than if expectations are rational. Our theoretical results thereby offer a possible explanation for the observed co-movement between the regional dispersion in inflation expectations and trade imbalances in the euro area, as shown in Figure 1.

By examining the model’s stability conditions, we find that a higher degree of home bias in expectations reduces the likelihood that expectations converge to a fixed point.\(^4\) We show that E-stability may not be guaranteed if expectations are home biased, even in parameter regions where monetary policy yields a determinate rational expectations equilibrium. Intuitively, since the home bias strengthens the real interest rate channel, inflation expectations are more likely to become unanchored. To guarantee an E-stable equilibrium, the central bank must adopt a more aggressive monetary stance with regards to inflation, beyond what is suggested by the Taylor-principle.

The issue of internal adjustment under monetary union has been studied, among others, by Angeloni and Ehrmann (2007), Deroose et al. (2008) and Allsopp and Vines (2010). Angeloni and Ehrmann (2007) confirm the pro-cyclical and counter-cyclical effects of, respectively, the real interest rate and real exchange rate on inflation differentials in the euro area. Interestingly, the authors mention the possibility of “a strong ‘home bias’ in the mechanism driving inflationary expectations in the national economies” that could amplify inflation differentials (Angeloni and Ehrmann, 2007, p. 8). Deroose et al. (2008) and Allsopp and Vines (2010)

\(^4\)Because we model the home bias in expectations as permanent, beliefs do not saddle down to the rational expectations equilibrium and so we define stability in expectations as convergence towards a so-called Restricted Perception Equilibrium or RPE (see Evans and Honkapohja, 2001, Chapters 3.6 and 13). Significant contributions on the stability properties of RPE models are Branch and Evans (2006a), Berardi (2007), Berardi and Duffy (2007) and Branch and Evans (2011).
focus on national policies and reforms that help increase market flexibility and strengthen
the real exchange rate channel. Sources of macroeconomic imbalances are studied, among
others, by Blanchard (2007), who investigates the role of nominal rigidities and financial con-
straints, Siena (2014), who argues that anticipated reductions in external borrowing costs
have been the main driver of euro area imbalances, and Jaccard and Smets (2017), who show
that cross-country differences in financial structures can result in long-term financial imba-
lances between regions of a monetary union. Whereas these studies assume expectations to
be rational, we introduce an alternative distortion arising from non-rational expectations.

There exists a large body of work on non-rational heterogeneous expectations and their
implications for policy (see e.g. Hommes, 2011, for an overview). Many of the theoretical
models in this field assume heterogeneity arises from differences in forecasting heuristics. For
instance, Branch and McGough (2009), Massaro (2013) and Gasteiger (2014) assume that
only a fraction of the population have rational expectations, while others are boundedly
rational. In Honkapohja and Mitra (2006), agents use different forecasting methods that
yield dispersed predictions about future economic outcomes, while Mankiw and Reis (2002)
develop a model of sticky information where expectations are updated infrequently by a
random, yet constant, fraction of the population. Only a few papers relate the issue of
heterogeneous expectations to the stability of monetary unions. Toroj (2010) and Bertasiute
et al. (2018), for example, find macroeconomic volatility in a monetary union to be higher
when agents use different forecasting rules than when expectations are rational, while Carlin
(2013) shows that countries with non-rational wage setters are more vulnerable to pro-cyclical
real interest rate effects than countries with rational agents.

A key difference between these studies and the present paper is that we allow learning
methods to be symmetric across countries, since it is not a priori clear why these should
be asymmetric. If expectations were to differ across countries, we believe a more natural
and straightforward explanation would be that agents use different information sets, simply
because agents are more likely to notice changes in local rather than foreign variables. In
this regard, our paper is related to the literature on rational inattention (Sims, 2003) and bounded rationality (Gabaix, 2014). For instance, in Gabaix (2014) agents’ inattention is source-dependent, meaning that some information is ignored if it is deemed unnecessary for the optimization problem at hand. Rather than building the micro-foundations that can explain the sources of inattention, which can be many, the primary goal of this paper is to study the interaction between home bias in expectations and macroeconomic imbalances in a monetary union. We provide a simple framework in which the role of this home bias can be easily studied.

The rest of the paper is structured as follows. In the next section, we describe how expectations in our model are formed and how they are subjected to home bias, and present the main building blocks of the model. We study the dynamics of the model in Section 3, while the E-stability analysis is performed in Section 4. Finally, Section 5 concludes.

2 A model with home biased expectations

The main contribution of this paper is to allow for home bias in expectations in an otherwise standard multi-country dynamic general equilibrium model. The idea is that agents, as they acquire information, focus more strongly on news about domestic variables than about foreign variables. Such home bias in the formation of expectations may arise due to limitations in the capacity to acquire and process information (see e.g. Simon, 1984, Sims, 2003, Gabaix, 2014 and Coibion et al., 2018). This may be especially relevant with regards to foreign information due to language barriers, unfamiliarity with certain data sources, uncertainty about data quality, etc. It might therefore be optimal to deliberately ignore some of the information available.

The idea of home bias in expectations has been studied most prominently in the finance literature, typically in connection to the observed home bias in asset portfolios (French and Poterba, 1991). Gehrig (1993), for instance, shows that a model in which domestic investors
observe signals on domestic firms’ returns with higher precision than foreign investors can explain why equity investments are tilted more towards domestic assets, over and beyond what can be explained by other factors such as capital market restrictions. Moreover, Van Nieuwerburgh and Veldkamp (2009) show that investors may want to magnify this particular informational advantage and learn more about domestic assets in order to achieve higher excess returns. Tille and van Wincoop (2014) study a related issue in a two-country general equilibrium model, in which agents receive private signals about future productivity shocks and where signals about domestic shocks are more precise. They show that the dispersion in information and expectations that results from these private signals greatly impacts cross-border capital flows. The informational advantage of locals with regards domestic assets has also been documented empirically, for instance by Portes and Rey (2005), Bae et al. (2008) and Leuz et al. (2008). Furthermore, Mondria et al. (2010) show that investors are generally more attentive to news about familiar countries and that home bias in equity holdings could be reduced substantially if attention was allocated equally across countries. Similarly, Huang (2015) shows that investors tend to underreact to foreign news and to information from more linguistically and culturally distant countries. Finally, Beneish and Yohn (2008) show that investor home bias persists, even when information processing costs are reduced through adoption of international reporting standards. This finding suggests that the attention bias of investors is likely to be highly rigid and invariant to policy changes. In this section, we present an adaptive learning model that features a similar type of attention bias with regards to foreign information.

2.1 An adaptive learning model

We consider a New Keynesian model for a monetary union that consists of two countries, Home (H) and Foreign (F), along the lines of Benigno (2004). Letting \( x_t \) be a vector of the model’s endogenous variables and \( v_t \) a vector of exogenous variables, the dynamics of a linearized version of the model can be summarized in canonical form by the following two
equations:

\[ Ax_t = \sum_{i=\{H,F\}} B_i \hat{E}_t^i x_{t+1} + C x_{t-1} + D v_t, \]  
\[ \quad v_t = \varrho v_{t-1} + \varepsilon_t, \]  
where \( A, B^i, C \) and \( D \) are coefficient matrices of appropriate sizes, \( \varrho \) a diagonal matrix with all eigenvalues inside the unit circle, and \( \varepsilon_t \) a vector of i.i.d. normal shocks with mean zero and variance \( \sigma^2 \). \( \hat{E}_t^i \) is the expectations operator used by residents of country \( i = \{H,F\} \). In what follows, we assume that expectations are the same across agents within the same country, yet may differ across countries. Moreover, the expectations operator satisfies the assumptions specified in Branch and McGough (2009), which facilitates aggregation and requires that the law of iterated expectations holds both at the individual and country level.

The Rational Expectations Equilibrium (REE) of the model above has the following form:

\[ x_t = \Lambda_1 x_{t-1} + \Lambda_2 v_t. \]  
Under the adaptive learning approach considered here, agents do not know the true coefficients matrices, \( \Lambda_1 \) and \( \Lambda_2 \), yet must obtain estimates using past data on domestic and non-domestic variables (see Evans and Honkapohja, 2001).\(^5\) We capture home bias in expectations parsimoniously by a single parameter that weighs down the non-domestic variables in agents’ information sets.

Specifically, let \( x_{d,t} \) denote a subset of \( x_t \) containing only domestic endogenous variables and \( x_{n,t} \) a subset containing non-domestic endogenous variables. The data vector can then be partitioned as \( x_t = [x_{d,t}, x_{n,t}]' \). Likewise, the exogenous variables can be partitioned as \( v_t = [v_{d,t}, v_{n,t}]' \). Using this transformation, we can write the household’s Perceived Law of

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\(^5\)In this paper, we follow the so-called ‘Euler equation learning’ approach, which assumes agents make consumption decisions to satisfy one-period ahead perceived Euler equations. Consequently, agents do not take into account future revisions of their beliefs when forming expectations. See also Evans and Honkapohja (2001) and Bullard and Mitra (2002).
Motion (PLM) as follows:

\[ x_t = \Lambda_{0,t-1}^i + \Lambda_{1,t-1}^i \begin{bmatrix} I & 0 \\ 0 & I\omega \end{bmatrix} \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix} + \Lambda_{2,t-1}^i \begin{bmatrix} I & 0 \\ 0 & I\omega \end{bmatrix} \begin{bmatrix} v_{d,t} \\ v_{n,t} \end{bmatrix}, \]

(4)

with \(0\) the zero matrix of conformable size, and where \(\omega \in [0, 1]\) denotes the home bias parameter.\(^6\) The PLM is used to make forecasts of \(x_t\), i.e. \(\hat{E}_t^i x_{t+1}\), which feed back into (1). The home bias parameter determines how much of the non-domestic information, \(\{x_{n,t}, v_{n,t}\}\), is used when performing these forecasts. In fact, since the model is linearized, \(x_t\) collects the percentage deviations of the endogenous variables from their respective steady states. Hence, \(\omega\) captures the degree of variation in non-domestic variables that is considered by agents when forming expectations. We interpret the home bias parameter as a parsimonious modeling alternative to more elaborate micro-founded theories, such as the concept of rational inattention of Sims (2003) or the sparse maximization approach of Gabaix (2014) in which the degree of inattention is chosen optimally for a given parameterized attention cost. As with these attention costs, we take the home bias parameter as given and remain agnostic with regards its sources, which can be many.\(^7\) Importantly, we assume \(\omega\) to be exogenous and time-invariant. Although one could imagine attention allocation to change endogenously over time, the finance literature discussed above suggests that home bias is

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6This approach of modeling home bias in expectations can be generalized by allowing for different home bias parameters (distinguishing between e.g. endogenous and exogenous variables, or even different individual variables) and by making the home bias parameter time dependent. However, we choose a single and time-invariant home bias parameter for expositional simplicity.

7One could, for instance, interpret the home bias parameter \(\omega\) as the proportion of agents that focus solely on domestic variables while forming expectations. These agents may, for instance, earn their income from the production of non-tradable goods only, leaving them less concerned about foreign economic developments. In that case, the aggregate data vector observed in country \(i\) is simply a weighted average of the information sets stemming from the tradable sector, \(x_{T,t}^i\), and the non-tradable sector, \(x_{NT,t}^i\), i.e.

\[
\tilde{x}_t^i = \omega x_{T,t}^i + (1 - \omega) x_{NT,t}^i.
\]

\[
= \omega \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix} + (1 - \omega) \begin{bmatrix} x_{d,t-1} \\ 0 \end{bmatrix}
\]

\[
= \begin{bmatrix} I & 0 \\ 0 & I\omega \end{bmatrix} \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix}.
\]
rooted deeply in investors’ expectations and persists even as technological advancements and reporting standards substantially reduce information frictions. Moreover, the constancy of the expectations bias is likely to be stronger for households and firms (who populate our model) than for investors, as the latter have more to gain financially from being fully informed about international economic events.

Denote $\Lambda_i^t \equiv [\Lambda_{0,t}, \Lambda_{1,t}, \Lambda_{2,t}]$ and $z_t \equiv [1, x_{t-1}, v_t]'$. Whenever new data becomes available, agents update their estimates for $\Lambda_i^t$ using the following updating equations:

\begin{align}
\Lambda_i^t &= \Lambda_i^{t-1} + \gamma_t \left( M_i^t \right)^{-1} \tilde{z}_i^t \left( \tilde{x}_i^t - \Lambda_i^{t-1} \tilde{z}_i^t \right)', \\
M_i^t &= M_i^{t-1} + \gamma_t \left( \tilde{z}_i^t \left( \tilde{z}_i^t \right)' - M_i^{t-1} \right).
\end{align}

The tildes above the data vectors, $\tilde{x}_i^t$ and $\tilde{z}_i^t$, denote the observed data. Equation (5) shows that the extent by which the coefficients are updated depends on the size of the forecast error, $\tilde{x}_i^t - \Lambda_i^{t-1} \tilde{z}_i^t$. The weight on the forecast error is determined by $M_i^t$, i.e. the moment matrix of $\tilde{z}_i^t$, and the gain parameter $\gamma_t$, which controls the speed with which agents learn. In what follows, we assume a constant gain, i.e. $\gamma_t = \gamma$ for all $t$, such that agents discount old data (with geometrically declining weights) and assign more weight to new information.\(^8\)

By limiting the amount of data used, the home bias parameter $\omega$ affects the moment matrix of the data, $M_i^t$, which in turn alters the way agents update their estimates of $\Lambda_i^t$ by (5) and (6). Hence, agents’ beliefs with regards economic dynamics are shaped by the degree of home bias in expectations. Moreover, due to the self-referential nature of the system, in which realized variables and expectations affect each other, the regressors in the PLM (4) are endogenous and so the Frisch-Waugh-Lovell theorem does not apply here. Hence, one

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\(^8\)Many papers that study macroeconomic dynamics using adaptive learning models adopt the constant gain assumption (e.g. Orphanides and Williams, 2005a, Milani, 2007, Branch et al., 2014, and Lubik and Matthes, 2016), as it tends to improve the empirical fit relative to models featuring a time-varying gain, which in turn are typically used to study asymptotic properties and convergence. Moreover, Malmendier and Nagel (2015) show empirically that a constant gain learning algorithm approximates quite closely the average learning-from-experience forecasts in a model with heterogeneous beliefs in which individuals follow a decreasing gain learning algorithm. Nakov and Nuño (2015) derive a similar result using a theoretical asset pricing model with overlapping generations.
cannot simply rewrite the updating equations (5)-(6) in such a way that the effect of the home bias parameter on the estimated coefficients is nullified. However, if the home bias would pertain only to the non-domestic *exogenous* variables $v_{n,t}$, and not to the endogenous variables $x_{n,t}$, then the Frisch-Waugh-Lovell theorem applies and the home bias parameter has no effect on the learning dynamics.\footnote{In the Appendix, we use a simple model to illustrate under which conditions the home bias parameter affects the learning process in this setup.}

### 2.2 Building blocks of the model

Each member state of the monetary union is populated by a continuum of households and firms. Home firms are indexed by $h \in [0, s)$ while Foreign firms are indexed by $f \in [s, 1]$, with $s \in [0, 1]$ measuring the relative size of Home. A supranational central bank sets monetary policy and targets union-wide aggregates.\footnote{A more detailed description of the model is provided in the Appendix.}

#### 2.2.1 Households

A representative household living in country $i$ chooses consumption, $c_{i,t}$, and labor supply, $n_{i,t}$, to maximize expected life-time utility:

$$\hat{E}_{i}^{\infty} \sum_{t=0}^{\infty} \beta^{t} z_{D,t}^{i} \left( \frac{ (c_{i,t}^{i})^{1-\sigma} - (n_{i,t}^{i})^{1+\varphi} }{1 - \sigma} \right),$$

with $\beta \in (0, 1)$ the discount factor, $\sigma > 0$ the coefficient of relative risk aversion and $\varphi > 0$ the inverse Frisch elasticity of labor supply. Labor is supplied only to domestic firms. $z_{D,t}^{i}$ is a demand shock that evolves according to a stationary AR(1) process that is known to all agents. Consumption is a composite of domestically produced final goods, $c_{i,t}^{i}$, and imported final goods, $c_{j,t}^{j}$, with $j = \{H, F\}$ and $i \neq j$:

$$c_{i,t}^{i} \equiv \left[ \left( 1 - \alpha^{i} \right)^{\frac{1}{n}} \left( c_{i,t}^{i} \right)^{\frac{n-1}{n}} + \left( \alpha^{i} \right)^{\frac{1}{n}} \left( c_{j,t}^{j} \right)^{\frac{n-1}{n}} \right]^{\frac{n}{n-1}},$$

\footnote{In the Appendix, we use a simple model to illustrate under which conditions the home bias parameter affects the learning process in this setup.}

\footnote{A more detailed description of the model is provided in the Appendix.}
where \( \eta \geq 1 \) denotes the trade elasticity and \( \alpha_i \) determines the degree of country openness. Specifically, \( \alpha^H \equiv (1 - s) \alpha \) and \( \alpha^F \equiv s \alpha \), with \( \alpha \in (0, 1) \) the import share in consumption.

The household earns a nominal wage \( W_i^i \) for each unit of labor supplied and domestic firm profits \( P_i^i \). Furthermore, households have access to a complete set of one-period Arrow-Debreu securities, \( B_i^i (\xi_{t+1}) \), each unit of which pays out one unit of currency conditional on state \( \xi_{t+1} \) occurring. The flow budget constraint faced by the household for every event \( \xi_t \) at any time \( t \) is given by

\[
P_i^i c_i^i + \sum_{\xi_{t+1}} Q_t (\xi_{t+1} \mid \cdot) B_i^i (\xi_{t+1}) = W_i^i n_i^i + B_{t-1}^i + P_i^i P_t^i, \tag{9}
\]

with \( P_{t,t}^i \) the producer price index (PPI), \( P^i_t \) the consumer price index (CPI) and \( Q_t (\xi_{t+1} \mid \cdot) \) the price of an Arrow-Debreu security. We assume initial wealth to be identical across agents, i.e. \( B_{-1}^i = B \forall i \). Subject to (9) and the transversality condition \( \lim_{T \to \infty} Q_{t,T} B_T^i = 0 \), the household maximizes the objective function (7) which yields the optimal labor supply condition:

\[
\left( n_i^i \right)^{\varphi} = w_i^i \left( c_i^i \right)^{-\sigma}, \tag{10}
\]

and the optimal intertemporal allocation of wealth between consumption and savings:

\[
1 = \beta \tilde{E}_t \left[ \frac{R_t}{\pi_{t+1}^i} \left( \frac{c_{t+1}^i}{c_t^i} \right)^{-\sigma} \frac{z_{D,t+1}^i}{z_{D,t}^i} \right], \tag{11}
\]

with \( w_i^i \equiv W_i^i / P_i^i \) the real wage rate, \( \pi_i^i \equiv P_i^i / P_{t-1}^i \) gross CPI inflation and \( R_t \) the gross nominal interest rate set by the central bank, satisfying \( 1 / R_t = \sum_{\xi_{t+1}} Q_t (\xi_{t+1}) \).

Combining (11) for both countries yields the ‘imperfect international risk-sharing condition’:\(^{11}\)

\[
\frac{z_{D,t+1}^H \left( \frac{c_{t+1}^H}{c_t^H} \right)^{-\sigma}}{z_{D,t}^H} = \frac{1}{q_t} \left\{ \frac{\tilde{E}_t^H \left[ z_{D,t+1}^H \left( \frac{c_{t+1}^H}{c_t^H} \right)^{-\sigma} / P_{t+1}^H \right] \}}{\tilde{E}_t^F \left[ z_{D,t+1}^F \left( \frac{c_{t+1}^F}{c_t^F} \right)^{-\sigma} / P_{t+1}^F \right] \}} \right\}, \tag{12}
\]

\(^{11}\)See the Appendix for a full derivation.
with the real exchange rate, \( q_t \), defined as the ratio between the Foreign and Home CPI, i.e.
\[
q_t \equiv \frac{P^F_t}{P^H_t}. \tag{13}
\]

The term in curly brackets on the right-hand side of (12) reflects the relative expected marginal utility of consumption across the two countries. If expectations are homogeneous, i.e. \( \tilde{E}^H_t = \tilde{E}^F_t \), then due to the assumption of complete asset markets and symmetric initial conditions, this term equals unity and perfect risk sharing is ensured through appropriate adjustments in the real exchange rate. However, under heterogeneous expectations, the expected marginal utility of consumption may differ across countries and perfect risk sharing breaks down. The greater is the degree of heterogeneity in expectations, the more distorted is the risk sharing condition.

In contrast to other models featuring heterogeneous expectations, we assume that agents adopt the same learning algorithm given by Equations (5) and (6). What may cause beliefs to differ across Home and Foreign agents is the inattention to non-domestic variables, as discussed in Section 2.1. Because learning algorithms are symmetric across agents, and because Home and Foreign shocks share the same distributional properties, neither the Home nor Foreign agent possesses a superior learning model. It thus follows that both type of agents make under- or overpredictions equally often (i.e. forecast errors are of equal magnitude) and, over time, no agent will take an infinite position in the Arrow-Debreu securities (see Xiong and Yan, 2009).\textsuperscript{12}

2.2.2 Firms and price setting

Differentiated goods \( y^i_t(z) \) are produced by monopolistic wholesale firms, indexed by \( z = \{h, f\} \), using the constant returns to scale production function \( y^i_t(z) = n^i_t(z) \). Following

\textsuperscript{12}Infinite positions could occur if, for instance, agents were to follow different learning algorithms, causing some agents to be more or less confident or optimistic about their beliefs. In this case, and under complete asset markets, wealth dynamics diverge across agents such that borrowing limits become binding (see Cogley et al., 2014, and Santoro, 2017).
Calvo (1983), firms can change their price only with probability $1 - \theta$. With probability $\theta \in (0, 1)$, prices are kept fixed at last period’s PPI. Firms set their price to maximize expected discounted profits, subject to a demand schedule, the production function and a condition that pins down the optimal demand for labor, i.e. $mc_i^t = \left( P_i^i / P_{it}^i \right) w_i^t$, with $mc_i^t$ denoting real marginal costs. The resulting optimal price, $P_i^t$, is determined by

$$P_i^t = \frac{\epsilon}{\epsilon - 1} \frac{\bar{E}_i^t \sum_{k=0}^{\infty} \theta^k Q_{t+k} (\xi_{t+k+1}^t) \left( P_{i,t+k}^i \right)^{1+\epsilon} y_{t+k}^i mc_{i+k}^i}{\bar{E}_i^t \sum_{k=0}^{\infty} \theta^k Q_{t+k} (\xi_{t+k+1}^t) \left( P_{i,t+k}^i \right)^{\epsilon} y_{t+k}^i}, \quad (14)$$

where $\epsilon > 1$ measures the elasticity of substitution between goods.

2.2.3 The central bank and monetary policy

A common central bank sets the gross nominal interest rate according to the following rule:

$$\frac{R_t}{\bar{R}} = \left( \frac{\pi MU, t}{\pi MU} \right)^{\phi_\pi} \left( \frac{y MU, t}{y MU} \right)^{\phi_y}, \quad (15)$$

where union-wide aggregates are weighted by country size, i.e. $x_{MU, t} \equiv sx_{i}^H + (1 - s) x_{i}^F$ for $x = \{\pi, y\}$, and variables without a $t$ subscript denote steady-state values. The monetary policy rule (15) is known to all agents.

2.2.4 Market clearing

Assuming the Law of One Price holds for all goods and using appropriate aggregators for national output, we can derive the goods market clearing condition for Home:

$$y_i^H = \left( \frac{P_{i,t}}{P_{d,t}} \right)^{-\eta} \left[ (1 - \bar{\alpha}^H) c_i^H + \frac{1 - s}{s} \bar{\alpha}^F q_i^H c_i^F \right], \quad (16)$$

and Foreign:

$$y_i^F = \left( \frac{P_{i,t}}{P_{d,t}} \right)^{-\eta} \left[ (1 - \bar{\alpha}^F) c_i^F + \frac{s}{1 - s} \bar{\alpha}^H q_i^H c_i^H \right]. \quad (17)$$
It is assumed that agents take prices as given and know that the goods market clearing conditions (16) and (17) need to be satisfied in equilibrium. Labor market clearing satisfies
\[ n^i_t = y^i_t D^i_t, \]
where \( D^i_t \) is a measure of price dispersion. Finally, we define Home’s trade balance as the difference between exports and imports:
\[
\begin{align*}
tb_{H,t} &= \alpha^F q_t \left( \frac{P_{H,t}}{P_t^H} \right)^{-\eta} c_t^F - \alpha^H q_t^{-\eta} \left( \frac{P_{F,t}}{P_t^F} \right)^{-\eta} c_t^H.
\end{align*}
\]
(18)

In what follows, we use \( tb_{H,t} \) as a proxy for macroeconomic imbalances within the monetary union.

From the perspective of Home (Foreign) agents, all variables with a superscript \( H \) (\( F \)) are considered domestic and therefore enter \( x_{d,t} \) in the PLM given by (4). On the other hand, variables with a superscript \( F \) (\( H \)) are considered non-domestic by Home (Foreign) agents, and so these enter \( x_{n,t} \). This implies that, for instance, the real exchange rate, whose dynamics are driven by both Home and Foreign prices, is imperfectly observed to the extent expectations are home biased.

### 2.3 Monetary union and the persistence of country-specific shocks

Before we study the implications of home bias in expectations for macroeconomic imbalances under monetary union, which is the main focus of the paper, we first discuss how being in a monetary union may cause country-specific shocks be more amplified, even under rational expectations.

According to Equation (15), the central bank only targets union-wide aggregates. Therefore, country-specific shocks may not always prompt a sufficiently strong monetary response insofar union-wide aggregates remain largely unchanged. This might be the case, for instance, if shocks are negatively correlated across countries or borne from relatively small regions within the union. It is this inability to tailor monetary policy to country-specific disturbances that allows temporary shocks to have long-lasting effects.
To better understand the peculiar nature of the propagation of shocks under monetary union, consider the linearized Home Euler equation:

$$\sigma \hat{c}_t^H = \sigma \hat{E}_t^H \hat{c}_{t+1}^H - \left( \hat{R}_t - \hat{E}_t^H \hat{\pi}_{t+1}^H \right) + (1 - \rho_D) \hat{z}_{D,t}^H,$$

where variables with a hat denote percentage deviations from steady state. The parameter $\rho_D \in (0, 1)$ measures the persistence of the Home demand shock $\hat{z}_{D,t}^H$. A temporary rise in $\hat{z}_{D,t}^H$ raises consumption, $\hat{c}_t^H$, and thereby aggregate output in Home, $\hat{y}_t^H$. Consequently, producer price inflation, $\hat{\pi}_{H,t}^H \equiv \hat{P}_t^H / \hat{P}_{t-1}^H$, rises by the firm’s linearized price-setting condition:

$$\hat{\pi}_{H,t}^H = \beta \hat{E}_t^H \hat{\pi}_{H,t+1}^H + \kappa \left( \varphi \hat{y}_t^H + \sigma \hat{c}_t^H + \hat{\alpha} \hat{q}_t \right),$$

with $\hat{\alpha}^H \equiv \bar{\alpha}^H / \left( 1 - \bar{\alpha}^H - \bar{\alpha}^F \right) \geq 0$ and $\kappa \equiv (1 - \theta) (1 - \beta \theta) / \theta > 0$. CPI inflation then rises as well by the condition $\hat{\pi}_t^H = \hat{\pi}_{H,t}^H + \hat{\alpha}^H (\hat{q}_t - \hat{q}_{t-1})$ which in turn leads to an increase in expected inflation and a fall in the ex-ante real interest rate, $\hat{R}_t - \hat{E}_t^H \hat{\pi}_{H,t+1}^H$. The lower real interest rate induces a further rise in household consumption, firm production, inflation and so on. We refer to this pro-cyclical channel as the real interest rate channel. Under monetary union, this channel is strengthened by the fact that the central bank does not adjust the nominal interest rate, $\hat{R}_t$, to stabilize inflation in a specific region of the union. Rather, the central bank is concerned only with inflation at the union-wide level. Therefore, even if short-lived, country-specific shocks can have strong and persistent effects on the economy, even under rational expectations. With backward-looking expectations, the link between realized and expected inflation is tightened which further strengthens the real interest rate channel.

Once inflation rises, the real exchange rate appreciates, i.e. $\hat{q}_t$ falls. Since a stronger exchange rate lowers exports to Foreign, firm production in Home falls which leads to de-
cline in producer price inflation by (20). In turn, the real interest rate rises which reduces consumption and, hence, inflation. This counter-cyclical channel is referred to as the *real exchange rate channel* and it ensures that the real interest rate is adjusted back to steady state. The strength of the real exchange rate channel is influenced by the degree of country openness, determined by $\alpha$, and price stickiness, which is governed by $\theta$. The more open the economy is to foreign trade, i.e. the higher is $\alpha$, the more sensitive are price-setting decisions to changes in the real exchange rate, and hence the stronger is the real exchange rate channel. Equivalently, the more flexible are prices, i.e. the lower is $\theta$, the faster do prices adjust for a given change in the real exchange rate. In both cases, macroeconomic convergence results more rapidly following the country-specific demand shock.

Note that the strength of the real interest rate channel relative to the real exchange rate channel also depends on the degree of home bias in expectations. If the home bias is strong, expectations do not respond much to changes in the real exchange rate, since real exchange rate dynamics are in part determined by foreign variables and agents ignore such variables. As a consequence, agents do not fully internalize real exchange rate fluctuations when making price-setting and consumption decisions, which weakens the real exchange rate channel. Thus, by strengthening the real interest rate channel relative to the real exchange rate channel, the home bias in expectations has the potential to amplify country-specific disturbances, over and above the amplification that arises from the central bank’s mandate of targeting union-wide aggregates.

3 Macroeconomic imbalances in the monetary union

We simulate macroeconomic imbalances in the monetary union by means of an asymmetric demand shock that raises consumption in Home and lowers consumption in Foreign. We assume that the shocks are perfectly negatively correlated and that the countries are of equal size, i.e. $s = 0.5$. These assumptions imply that union-wide aggregates and the
monetary policy stance remain unchanged. Since monetary policy is essentially passive, the asymmetric demand shock has a persistent effect through the channels described in Section 2.3, thereby generating large and prolonged macroeconomic imbalances.\footnote{The assumption of a perfectly negatively correlated demand shock is adopted here for illustrative purposes only as it allows us to neutralize the role of monetary policy, which is the empirically relevant case for member states of a monetary union that face country-specific shocks. Nevertheless, all our results go through if demand shocks are uncorrelated or even imperfectly positively correlated. However, different results obtain when shocks are perfectly positively correlated, as this transforms the model into a single closed economy in which home bias in expectations has no effect.}

3.1 **Strategy and calibration**

We compute the impulse responses to the asymmetric demand shock using the method suggested by Eusepi and Preston (2011). In particular, we simulate the model twice, each round exposing the economy to a series of small uncorrelated shocks, randomly drawn from a normal distribution, for \( T \) periods. During each simulation round, agents update their beliefs periodically according to the learning algorithm described in Section 2. In line with the literature, we assume that agents’ initial beliefs coincide with those under rational expectations.\footnote{We initialize beliefs at the REE to make our results more comparable with existing studies. Also, as we discuss below, there may be multiple (stable) RPE’s which makes initialization at the RPE somewhat arbitrary.} In one of the two rounds, the economy is hit by the asymmetric demand shock at time \( T - K \), where \( K \) is the impulse response horizon. The impulse responses are then computed as the difference between the two simulation rounds from time \( T - K \) to \( T \). This approach allows us to track the response of the economy to the shock while agents are still updating their beliefs. We set \( T = 500 \) and \( K = 40 \).

The main parameter of interest is the home bias parameter, denoted by \( \omega \in [0,1] \). Since the literature does not provide a reasonable estimate of \( \omega \), we set the home bias parameter at 0.1 as a baseline. We do, however, consider a wide range of alternative values for \( \omega \) as a robustness and to illustrate the effects of home bias in expectations on macroeconomic dynamics. With regards the gain parameter, we set \( \gamma \) equal to 0.01.\footnote{Using, instead, a decreasing gain of \( \gamma = 1/t \) does no qualitative harm to our results.} This value lies between existing empirical findings, with somewhat higher estimates reported for professional fore-
Table 1: Benchmark calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Coefficient of relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Import-share of consumption</td>
<td>0.25</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of non-price adjustment</td>
<td>0.75</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Trade elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Monetary response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Monetary response to output</td>
<td>0.25</td>
</tr>
<tr>
<td>$s$</td>
<td>Relative size of Home</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>AR(1) coefficient of the demand shock</td>
<td>0.9</td>
</tr>
</tbody>
</table>

casters (Branch and Evans, 2006b) and lower estimates for households (Pfajfar and Santoro, 2010). For robustness, we also consider alternative values for $\gamma$.

The remaining model parameters are calibrated at a quarterly frequency. An overview of the benchmark calibration can be found in Table 1. Many parameters are assigned values that are common in the literature. As discussed in Section 2.3, the parameters that govern the probability of non-price adjustment and country openness, i.e. $\theta$ and $\alpha$ respectively, have a strong influence on the relative strengths of the real interest rate and real exchange rate channels. We set $\theta$ equal to the conventional value of 0.75, implying an average fixed-price duration of 1 year, while $\alpha$ is set to 0.25, which matches the average import-to-output ratio of bigger euro area countries like France, Italy and Spain in pre-crisis years.

### 3.2 Impulse response functions

The responses of the endogenous variables to the asymmetric demand shock are displayed in Figure 2. The solid lines are the responses under rational expectations, the dashed lines the responses under adaptive expectations without home bias (homogeneous expectations), and the dotted lines the responses under home biased expectations. Recall that the demand shock is positive in Home and negative in Foreign. Therefore, consumption rises in Home, which leads to an increase in Home CPI inflation and the Home trade deficit, whereas Foreign consumption and inflation fall.
Figure 2: Responses to an asymmetric demand shock

Note: Responses are expressed in percentage deviations from steady state. The demand shock is perfectly negatively correlated across the two countries, with Home (Foreign) experiencing a positive (negative) shock.
Since the shock has opposing effects across countries, union-wide inflation and output remain unchanged and so the nominal interest rate set by the central bank does not respond to the shock. Consequently, the real interest rate falls in Home, which further stimulates Home consumption and inflation. As discussed in Section 2.3, this pro-cyclical real interest rate channel arises from the household’s Euler equation that links expected consumption growth to the ex-ante real interest rate. In Foreign, the opposite occurs and economic activity is suppressed by a fall in Foreign inflation (expectations) and an increase in the real interest rate. Thus, even under rational expectations, the asymmetric demand shock generates persistent macroeconomic imbalances within the monetary union, due to the real interest rate channel.

Meanwhile, the rise in Home inflation leads to an appreciation of the real exchange rate, which dampens the economic boom through a contraction in aggregate demand, thereby improving the trade balance. Conversely, the real exchange rate depreciates from the perspective of Foreign, which helps recover economic activity. Hence, the destabilizing real interest rate channel is offset, endogenously, through a stabilizing real exchange rate channel that aids in restoring the trade imbalance within the monetary union.

Under home biased expectations, the responses of the endogenous variables are more pronounced and more persistent than under rational and unbiased expectations. In generating the impulse responses under home biased expectations, we set $\omega$ equal to 0.1. Other values for $\omega$ are considered below. Due to the home bias, agents do not fully internalize the relative change in domestic versus foreign prices and, as a consequence, underestimate the effects of the real exchange rate on the domestic economy. The home bias in expectations thus weakens the real exchange rate channel. Moreover, as agents focus more on domestic economic conditions, inflation expectations in Home (Foreign) remain high (low), and the real interest rate low (high), for a much longer period than in the absence of home bias in expectations, which strengthens the real interest rate channel. Due to the shift in the relative strengths of the two channels, the demand shock has a more amplified effect under
home biased expectations than if expectations were rational or free from home bias. This is reflected by a sharper adjustment of the real exchange rate and larger macroeconomic imbalances following the demand shock.

The main message conveyed by these impulse response functions is that real interest rate differentials across member states of a monetary union can lead to larger and more prolonged macroeconomic imbalances if agents, while forming their expectations, rely more on information inferred from domestic variables, while ignoring foreign and union-wide variables. The amplification of the effects of real interest rate differentials is due to a pro-cyclical real interest rate channel that undercuts a stabilizing real exchange rate channel. To overcome the home bias in agents' expectations, a more extreme adjustment in nominal prices is required, given that, under monetary union, the nominal exchange rate is fixed and monetary policy tools are inefficient in the face of asymmetric shocks.

In the left panel of Figure 3, we analyze further the implications of home bias in expectations for macroeconomic imbalances. We continue to use Home’s trade balance as a measure of macroeconomic imbalance and focus on its response to the asymmetric demand shock under alternative values for the home bias parameter $\omega$. The results suggest that, for a greater degree of home bias in expectations, macroeconomic imbalances are larger and macroeconomic adjustment is slower. The intuition follows form our discussion above: the home bias causes agents to ignore relative international price changes and underestimate the impact of the real exchange rate on economic dynamics, which weakens the real exchange rate channel and strengthens the real interest rate channel.

In the middle panel of Figure 3 we plot the response of Home’s trade balance to an asymmetric demand shock for different values of the gain parameter, $\gamma$, while keeping the home bias parameter fixed at $\omega = 0.1$. As is evident from the figure, the larger is the gain parameter, the greater is the resulting macroeconomic imbalance within the monetary union for a given asymmetric demand shock. In particular, a higher gain implies that agents assign a greater weight to current forecast errors when updating their beliefs. Because forecast
Figure 3: Response of Home’s trade balance to an asymmetric demand shock

Note: $\omega \in [0,1]$ measures the home bias in expectations, $\gamma > 0$ is the gain parameter in Equations (5) and (6), and $\alpha \in [0,1]$ determines the import-share in household consumption.

errors tend to be larger when beliefs are home biased, shocks are amplified the higher is $\gamma$. At the same time, however, a higher gain also implies that agents infer more quickly the stabilizing role of the real exchange rate, thus inducing a more rapid macroeconomic adjustment in the medium to long run.

In the right panel of Figure 3, the response of Home’s trade balance to the asymmetric demand shock is generated under different assumptions about country openness, which is governed by the import share in household consumption $\alpha$, while again keeping the home bias parameter constant at $\omega = 0.1$. The results from this figure convey the following message: the more the economy is exposed to foreign trade, i.e. the higher is $\alpha$, the stronger is the effect of the real exchange rate on economic conditions and therefore the quicker are agents to discover the counteracting forces of the real exchange rate. This result can also be interpreted differently by noting that consumer prices are a weighted average of both domestic and foreign producer prices. Evidently, the greater is $\alpha$, the more CPI inflation contains information on foreign price developments. Thus, even if foreign prices are largely
Note: The horizontal axis measures the difference between the Home agent’s inflation expectations, $\tilde{E}_H^t \pi_{t+1}^H$, and the Foreign agent’s inflation expectations, $\tilde{E}_F^t \pi_{t+1}^F$, at period $t$.

ignored directly by agents, they are indirectly inferred through changes in CPI inflation, and more so the more open is the economy to foreign trade. Consequently, despite the presence of home bias in expectations, large and prolonged macroeconomic imbalances following an asymmetric demand shock can be avoided if home bias in consumption is limited.

The connection between, on the one hand, regional dispersion in expectations and, on the other hand, macroeconomic imbalances in a monetary union was alluded to in Figure 1 in the Introduction, where we plotted regional dispersion in survey measures for inflation expectations against the dispersion in trade balance ratios across a group of euro area countries. We now use our model to see if we can recover this empirical observation. Particularly, we simulate the model for $T$ periods, again subjecting the economy to a series of randomly drawn, uncorrelated normal shocks. From the simulation, we save the difference in inflation expectations between Home and Foreign agents as well as Home’s trade balance for each period $t = 1, 2, ..., T$ and plot the two series against each other.
In the left panel of Figure 4, we show the results from the simulation without home bias in expectations. In this case, the cross-country dispersion in inflation expectations and the trade balance seem virtually unrelated. However, when allowing for home bias in expectations, as we do in the right panel, we see that more dispersed expectations are associated with greater trade imbalances, consistent with what is observed in the data. This exercise illustrates that it is the home bias in expectations that amplifies macroeconomic imbalances, and not solely the cross-country dispersion in expectations.

4 Stability and implications for monetary policy

In this section, which consists of two parts, we investigate how home bias in expectations affects the ability of the central bank to deliver a stable and determinate equilibrium in the monetary union. In the first part, we derive the conditions for E-stability of the model. We focus on the notion of strong E-stability in which agents, when making their forecasts, include both state and control variables in the data vector \( x_t \) and, therefore, are likely to over-parameterize their forecasting model. Furthermore, we use the concept of decreasing (rather than constant) gain learning, as it allows us to consider the possibility of convergent beliefs. In the second part, we perform numerical simulations to illustrate how the requirements for monetary policy to guarantee E-stability change under different degrees of home bias in expectations. We use the insights from the stability analysis to illustrate how monetary policy can mitigate the pro-cyclical effects of the real interest rate channel and promote macroeconomic stability.

4.1 The Restricted Perception Equilibrium

To derive the requirements for E-stability, we start by inserting agents’ PLM (4), iterated one period ahead, into the state-space representation of the model, given by (1), after multiplying
by $A^{-1}$ from the left, to find the Actual Law of Motion (ALM):

$$x_t = \left[ \sum_{i=\{H,F\}} F^i \left( I + \Lambda_1^i \Omega_x^i \right) \Lambda_0^i, \sum_{i=\{H,F\}} F^i \left( \Lambda_1^i \Omega_x^i \right)^2 + G, \sum_{i=\{H,F\}} F^i \left( \Lambda_1^i \Omega_x^i \Lambda_2^i + \Lambda_2^i \theta \right) \Omega_v^i + H \right] \left[ \begin{array}{c} 1 \\ x_{t-1} \\ v_t \end{array} \right]$$

(21)

with $F^i \equiv A^{-1} B^i$, $G \equiv A^{-1} C$, $H \equiv A^{-1} D$, and where $\Omega_k^i$, with $k = \{x,v\}$, are diagonal matrices of conformable sizes with either ones or $\omega$’s on the diagonal. These matrices determine by how much non-domestic variables and shocks are weighted down in agents’ PLM. Note that the location of the home bias parameter $\omega$ in $\Omega_k^i$ differs depending on whether expectations from Home or Foreign agents are considered as they perceive different variables as non-domestic.

The assumption of a potential home bias in agents’ expectations features a crucial departure from the standard adaptive learning literature, namely that agents in country $i$ observe only an imperfect version of the ALM, which is equal to Equation (21) multiplied by $\Omega_x^i$. Accordingly, agents face the following mapping from their PLM to their observed ALM:

$$T^i \left( \Lambda_0^i, \Lambda_2^i \Omega_x^i, \Lambda_2^i \Omega_v^i \right) = \left[ \begin{array}{c} \Omega_x^i \left[ \sum_{j=\{H,F\}} F^j \left( I + \Lambda_1^j \Omega_x^j \right) \Lambda_0^j \right] \\ \Omega_x^i \left[ \sum_{j=\{H,F\}} F^j \left( \Lambda_1^j \Omega_x^j \right)^2 + G \right] \\ \Omega_x^i \left[ \sum_{j=\{H,F\}} F^j \left( \Lambda_1^j \Omega_x^j \Lambda_2^j + \Lambda_2^j \theta \right) \Omega_v^j + H \right] \end{array} \right]'$$

(22)

If domestic and non-domestic variables are weighted differently in agents’ information sets, i.e. if $\omega < 1$, then the coefficients of the PLMs across countries are mapped into different observed ALMs, which implies that, in general, $\Lambda^H \neq \Lambda^F$. As a result, expectations differ across countries in steady state which prevents the economy from converging towards the REE. Instead, the economy saddles on an RPE. A necessary condition for the existence of such an RPE is that the mapping function, $T^i (\Lambda^i)$, has a fixed point, i.e. $\hat{\Lambda}^i = [\hat{\Lambda}_0^i, \hat{\Lambda}_1^i, \hat{\Lambda}_2^i]$. 

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satisfying

$$
\hat{\Lambda}_0^i = \Omega_x^i \left[ \sum_{j=\{H,F\}} F^j \left( I + \hat{\Lambda}_1^i \Omega_x^j \right) \hat{\Lambda}_0^i \right],
$$

(23)

$$
\hat{\Lambda}_1^i \Omega_x^j = \Omega_x^i \left[ \sum_{j=\{H,F\}} F^j \left( \hat{\Lambda}_1^i \Omega_x^j \right)^2 + G \right],
$$

(24)

$$
\hat{\Lambda}_2^i \Omega_v^j = \Omega_x^i \left[ \sum_{j=\{H,F\}} F^j \left( \hat{\Lambda}_1^i \Omega_x^j \hat{\Lambda}_2^i + \hat{\Lambda}_2^i \rho \right) \Omega_v^j + H \right].
$$

(25)

Because Equation (24) includes a quadratic matrix expression, multiple solutions for \( \hat{\Lambda}_1^i \) may exist. On the other hand, \( \hat{\Lambda}_2^i \) is uniquely pinned down by Equation (25) for any given \( \hat{\Lambda}_1^i \). Further, note that the only solution for \( \hat{\Lambda}_0^i \) is the zero vector.

The relevant ordinary differential equations that govern the E-stability properties of any RPE are derived in Appendix B and given by

$$
d\Lambda^i/d\tau = \varpi^i \left[ T^i \left( \Lambda_{i-1}^H, \Lambda_{i-1}^F \right) \left( \Omega_x^i \right)' - \left( \Omega_v^i \right)' \Lambda_{i-1}^i \right],
$$

(26)

with

$$
\Omega^i \equiv \begin{bmatrix} I & 0 & 0 \\ 0 & \Omega_x^i & 0 \\ 0 & 0 & \Omega_v^i \end{bmatrix},
$$

and where \( \varpi^i \) is defined as the asymptotic speed of adjustment in the learning algorithm, as described in Honkapohja and Mitra (2006). Since we assume agents in each country apply the same type of learning algorithm, we have \( \varpi^H = \varpi^F = \varpi \). Further, denote any possible RPE that satisfies the conditions (23)-(25) by \( (\hat{\Lambda}_H, \hat{\Lambda}_F) \). Then, by linearizing around the RPE and vectorizing the right-hand side of the differential equations (26), one obtains

$$
\begin{bmatrix}
\frac{d\text{vec}(\Lambda^H)}{d\tau} \\
\frac{d\text{vec}(\Lambda^F)}{d\tau}
\end{bmatrix} = \varpi \Xi \begin{bmatrix} \text{vec} (\Lambda^H) \\ \text{vec} (\Lambda^F) \end{bmatrix},
$$

(27)

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where \( \tau \) expresses ‘notional time’ as in Evans and Honkapohja (2001) and \( \Xi \) is given by

\[
\Xi = \begin{bmatrix}
\left( \Omega_z^H \otimes I \right) D_{\Lambda^H} T - I \otimes \left( \Omega_z^H \right)' & \left( \Omega_z^H \otimes I \right) D_{\Lambda^F} T \\
\left( \Omega_z^F \otimes I \right) D_{\Lambda^H} T & \left( \Omega_z^F \otimes I \right) D_{\Lambda^F} T - I \otimes \left( \Omega_z^F \right)'
\end{bmatrix}, \quad (28)
\]

with

\[
D_{\Lambda} T \left( \hat{\Lambda}^H, \hat{\Lambda}^F \right) = \begin{bmatrix}
\Omega_z^i F^i \left( I + \hat{\Lambda}_1^i \Omega_z^i \right) & \left( \Omega_z^i \hat{\Lambda}_1^i \right)' \otimes \Omega_z^i F^i & 0 \\
0 & \left( \Omega_z^i \hat{\Lambda}_1^i \Omega_z^i \right)' \otimes \Omega_z^i F^i & 0 \\
0 & \left( \Omega_z^i \hat{\Lambda}_2^i \Omega_z^i \right)' \otimes \Omega_z^i F^i & \left( \Omega_z^i \right)' \otimes \Omega_z^i F^i \end{bmatrix}. \quad (29)
\]

It follows that, under home biased expectations, E-stability of the RPE requires all eigenvalues of \( \Xi \) have negative real parts (Evans and Honkapohja, 2001). Next, we investigate the ability of monetary policy to deliver E-stability of the RPE under different degrees of home bias in expectations.

### 4.2 Monetary requirements for (E-)stability

Figure 5 shows the conditions for E-stability discussed earlier as a function of the monetary policy parameters \( \phi_{\pi} \in [0, 3] \), which governs the monetary response to inflation, and \( \phi_{y} \in [0, 3] \), which governs the monetary response to output. In the white area, monetary policy yields stable and unique rational expectations equilibria. These equilibria are also E-stable, i.e. learnable, RPEs. In the dark gray area, equilibrium is indeterminate, yet the RPE is E-stable. In the light gray area, the REE is determinate, whereas the RPE is not E-stable. Finally, the black region indicates the parameter space in which monetary policy is unable to deliver a determinate REE nor an E-stable RPE.

In the upper-left panel of Figure 5, expectations are free from home bias. In this case, we
Figure 5: Determinacy and E-stability for different degrees of home bias in expectations

Note: The figure shows the conditions for E-stability and determinacy as a function of the monetary policy parameters $\phi_\pi$ and $\phi_y$, and conditional on different degrees of home bias in expectations. White = determinate Rational Expectations Equilibrium (REE) and E-stable Restricted Perception Equilibrium (RPE); dark gray = indeterminate REE and E-stable RPE; light gray = determinate REE and E-unstable RPE; black = indeterminate REE and E-unstable RPE.
find that the Taylor-principle is a sufficient condition for both determinacy and E-stability. In other words, the central bank must respond to increases (decreases) in inflation by raising (lowering) the nominal interest rate by more than one for one, which requires $\phi_\pi > 1$. When the monetary policy response to output is sufficiently strong, $\phi_\pi$ can even be set slightly below unity (see e.g. Clarida et al., 1999).

The remaining panels of Figure 5 show that the E-stability requirements for monetary policy change when expectations are home biased. In fact, the greater is the degree of home bias in expectations, the higher is the likelihood of ending up in regions characterized by E-unstable RPEs. This is reflected by the expansion of the light-gray region as the home bias in expectations increases (i.e. as $\omega$ falls). To guarantee an E-stable equilibrium under home biased expectations, the central bank must respond more aggressively to changes in inflation or output (or both). Thus, the monetary policy trade-off between inflation and output stabilization worsens under home biased expectations.

These results are closely related to the Walters critique (Walters, 1994). Alan Walters, in his plea against UK membership of the European Monetary System, argued that monetary unions are inherently unstable due to the inability of member states to, unilaterally, offset country-specific shocks through an adjustment in the nominal interest rate or nominal exchange rate. Accordingly, countries belonging to a monetary union would be more susceptible to self-strengthening inflationary or deflationary cycles than countries operating under flexible exchange rates. Those who challenge the Walters critique point to the stabilizing effects of the real exchange rate channel that, at least in the long run, dominate the pro-cyclical effects of the real interest rate channel. The impulse responses shown in Section 3 indicate that both channels are relevant for macroeconomic stability and that the relative strengths of the two channels depend on the degree of home bias in expectations.

The results from the E-stability analysis suggest that monetary unions may indeed be more likely to be unstable, thereby supporting the Walters critique, yet only insofar expectations are home biased. If the home bias in expectations does not exist, we find that
the stability and determinacy requirements for monetary policy are not that different from those pertaining to economies outside monetary union. Yet, if expectations are subject to home bias, stability of the union is threatened. In that case, to avoid unstable macroeconomic outcomes, the central bank must adopt a more aggressive stance towards inflation and output.

To see whether a more aggressive monetary policy stance helps mitigate macroeconomic imbalances under home biased expectations, we again examine the effects of an asymmetric demand shock on Home’s trade balance for $\omega = 0.1$ and different values of the monetary policy parameters $\phi_\pi$ and $\phi_y$. The results are shown in Figure 6.

According to the figure, a more aggressive response of the central bank to changes in either inflation (i.e. higher values for $\phi_\pi$, see left panel) or output (higher values for $\phi_y$, right panel) reduces the trade balance’s response to the asymmetric demand shock. Recall that,
following the shock, the nominal interest rate is left unchanged as union-wide aggregates remain constant. However, different monetary policy stances still can deliver different macroeconomic dynamics, even without changing the nominal interest rate following the shock, due to their effect on agents’ expectations before the shock hits the economy. For instance, following an increase in aggregate demand, inflation expectations will remain closer to the inflation target if agents experienced a central bank that, in the past, was more hawkish rather than dovish. Therefore, a more aggressive monetary policy stance helps weaken the feedback between inflation expectations and the real interest rate, thereby also weakening the real interest rate channel and allowing the real exchange rate to stabilize economic conditions more promptly.

Although the benefits of having anchored inflation expectations, in terms of promoting macroeconomic stability, are well understood, the results shown here provide an additional argument in favor of tightly anchored inflation expectations that applies particularly to monetary unions: not only does a credible inflation target prevent expectations from drifting too far from steady state, it also helps prevent expectations from diverging too far across regions within the union following country-specific shocks.

5 Conclusion

Although the importance of expectations for policy design and evaluation is broadly acknowledged in the literature, attention is often focused on (rational) expectations at the aggregate level, thereby ignoring potentially important differences in expectations across agents. This approach does not seem appropriate when studying asymmetric shocks under monetary union, where forces that provoke divergences in economic dynamics and expectations across agents from different member countries are likely to be strong.

In this paper, we examined the implications of home bias in expectations for the size and duration of macroeconomic imbalances following an asymmetric demand shock in a two-
country monetary union. We found that home bias in expectations aggravates pro-cyclical real interest rate effects, causing macroeconomic imbalances be more pronounced. In particular, the expectations bias shifts agents’ attention from international price changes to domestic real interest rates, which hampers macroeconomic adjustment through the real exchange rate channel. We further showed that, for a higher degree of home bias in expectations, monetary policy must take a more aggressive stance towards inflation and output to anchor inflation expectations.

It is well established that cross-country inflation differentials under monetary union may reflect, at least in part, the endogenous response of the real exchange rate to asymmetric shocks. In fact, such a self-correcting mechanism forms an integral part of the smooth functioning of monetary unions. Hence, it is not always clear whether policymakers should care about dispersion in inflation rates across member countries. However, the results presented in this paper point towards a potential risk embedded in divergent inflationary dynamics under monetary union that is related to the way agents form expectations. To distinguish between regional economic disparities that are either temporary, and part of a necessary adjustment process, or structural, and potentially unstable, policymakers should monitor expectations at both the aggregate and national level. Internalizing inter-regional dispersion into the monetary policy strategy, then, is warranted insofar expectations are highly sensitive to past domestic economic conditions and ignorant of international spillover effects. Alternatively, fiscal policy could be employed to offset the bias in expectations at the national level, for instance by changing import tariffs or export subsidies to replicate the required real exchange rate adjustment. Whether such policies are able to fully undo the effects of the home bias in expectations is a question we leave for future research.
References


A The linear model

In this section, we present the linear version of the model. The model is linearized around a deterministic, zero-inflation steady state. Let variables with a hat denote the percentage deviation of the corresponding variable from its steady-state value, i.e. \( \hat{x}_t = (x_t - x) / x \) for any generic variable \( x_t \). A scaled-down version of the linear model then consists of the following equations.

First, the Euler equation for the Home country and the imperfect international risk sharing condition:

\[
\sigma \hat{c}_t^H = \sigma \tilde{E}_t^H \hat{c}_{t+1}^H + (1 - \rho_D) \hat{z}_{D,t}^H, \\
\sigma (\hat{c}_t^F - \hat{c}_t^H) = \sigma (\tilde{E}_t^F \hat{c}_{t+1}^F - \tilde{E}_t^H \hat{c}_{t+1}^H) + (1 - \rho_D) \hat{z}_{D,t}^F - \hat{z}_{D,t}^H, \\
\]

where the demand shock, \( \hat{z}_{D,t}^i \), evolves according to

\[
\hat{z}_{D,t}^i = \rho_D \hat{z}_{D,t-1}^i + \varepsilon_{D,t}^i, \\
\]

with \( i = \{H, F\} \), \( \rho_D \in [0, 1] \) and \( \varepsilon_{D,t}^i \sim \mathcal{N}(0, \sigma^2) \).

Second, the resource constraints of Home and Foreign:

\[
\hat{y}_t^H = \left(1 - \alpha^H\right) \frac{c^H}{y^H} \hat{c}_t^H + \frac{1 - s}{\alpha^F} \frac{c^F}{y^F} \hat{c}_t^F + \Theta_H \hat{q}_t, \\
\hat{y}_t^F = \left(1 - \alpha^F\right) \frac{c^F}{y^F} \hat{c}_t^F + \frac{s}{\alpha^H} \frac{1 - s}{y^H} \hat{c}_t^H - \Theta_F \hat{q}_t, \\
\]

with

\[
\Theta_H \equiv \eta^H \frac{\alpha^H}{\alpha^F} \left[ \alpha^H \left(1 - \alpha^H\right) \frac{c^H}{y^H} + \left(1 - \alpha^F\right) \frac{1 - s}{\alpha^F} \frac{c^F}{y^H} \right], \\
\Theta_F \equiv \eta^F \frac{\alpha^F}{\alpha^H} \left[ \alpha^F \left(1 - \alpha^F\right) \frac{c^F}{y^F} + \left(1 - \alpha^H\right) \frac{s}{\alpha^H} \frac{c^H}{y^F} \right].
\]
\[ \hat{\alpha}_i \equiv \alpha_i / \left( 1 - \alpha_H - \alpha_F \right). \]

Third, the conditions for Home and Foreign CPI inflation and a condition that governs real exchange rate dynamics:

\begin{align*}
\hat{\pi}_H^t &= \hat{\pi}_H^{H,t} + \hat{\alpha}_H \left( \hat{q}_t - \hat{q}_{t-1} \right), \\
\hat{\pi}_F^t &= \hat{\pi}_F^{F,t} - \hat{\alpha}_F \left( \hat{q}_t - \hat{q}_{t-1} \right), \\
\hat{q}_t &= \hat{q}_{t-1} + \hat{\pi}_F^t - \hat{\pi}_H^t.
\end{align*}

Fourth, the Home and Foreign New Keynesian Phillips curves:

\begin{align*}
\hat{\pi}_H^{H,t} &= \beta \hat{E}_t^H \hat{\pi}_H^{H,t+1} + \kappa \left( \varphi \hat{y}_t^H + \sigma \hat{c}_t^H + \hat{\alpha}_H \hat{q}_t \right), \\
\hat{\pi}_F^{F,t} &= \beta \hat{E}_t^F \hat{\pi}_F^{F,t+1} + \kappa \left( \varphi \hat{y}_t^F + \sigma \hat{c}_t^F - \hat{\alpha}_F \hat{q}_t \right).
\end{align*}

And finally, the monetary policy rule of the central bank:

\[ \hat{R}_t = \phi_{\pi} \left[ s \hat{\pi}_H^t + (1-s) \hat{\pi}_F^t \right] + \phi_y \left[ s \hat{y}_t^H + (1-s) \hat{y}_t^F \right], \]

which is perfectly observed by all agents in the monetary union.

After substituting (40) in (30), the state-space representation of the model is given by Equations (1) and (2) in Section 2.1, with \( x_t \equiv [x_t^H, x_t^F, \hat{q}_t]' \), \( x_t^i \equiv [\hat{g}_t^i, \hat{c}_t^i, \hat{n}_t^i, \hat{\pi}_t^i]' \) and \( u_t \equiv [\hat{z}_{D,t}^H, \hat{z}_{D,t}^F]' \). From the perspective of Home, \( x_t^H \) enters the domestic subset \( x_{d,t} \), while \( x_t^F \) and \( \hat{q}_t \) enter the non-domestic subset \( x_{n,t} \) (see Equation [4]). From the perspective of Foreign, \( x_t^F \) enters \( x_{d,t} \), while \( x_t^H \) and \( \hat{q}_t \) enter \( x_{n,t} \).
B Derivation of Equation (26)

To derive the ordinary differential equation (26), first consider (5) and substitute in for \( \tilde{x}_t^i \) by multiplying (21) by \( \Omega_x^i \) to find

\[
\Lambda_{t-1}^i + \gamma_t \left( M_t^i \right)^{-1} \tilde{z}_t^i \left[ \Omega_x^i T \left( \Lambda_{t-1}^H, \Lambda_{t-1}^F \right) z_t - \Lambda_{t-1}^i \tilde{z}_t^i \right]'
\]

\[
= \Lambda_{t-1}^i + \gamma_t \left( M_t^i \right)^{-1} \Omega_x^i z_t \left[ \Omega_x^i T \left( \Lambda_{t-1}^H, \Lambda_{t-1}^F \right) z_t - \Lambda_{t-1}^i \Omega_x^i z_t \right]'
\]

\[
= \Lambda_{t-1}^i + \gamma_t \left( M_t^i \right)^{-1} \Omega_x^i z_t \left\{ \left[ \Omega_x^i T \left( \Lambda_{t-1}^H, \Lambda_{t-1}^F \right) - \Lambda_{t-1}^i \Omega_x^i \right] z_t \right\} '
\]

Note that

\[
E \left( M_t^i \right)^{-1} \Omega^i z_t z_t' = \left[ \begin{array}{c}
\left\{ \Omega_x^i \left[ \sum_{i=\{H,F\}} \dfrac{F^i}{(I + A^i \Omega_x^i) A^i_0} - A^i_0 \right] \right\}'
\\
\left\{ \Omega_x^i \left[ \sum_{i=\{H,F\}} \dfrac{F^i}{(I + A^i \Omega_x^i)^2 + G} - \Omega_x^i A^i_0 \right] \right\}'
\\
\left\{ \Omega_x^i \left[ \sum_{i=\{H,F\}} \dfrac{F^i}{(I + A^i \Omega_x^i)^2 + G} - \Omega_x^i A^i_0 \right] \right\}'
\end{array} \right]
\]

Let \( E [zz'] \equiv \lim_t E [z_t z_t'] \). Then, we obtain the following ODEs:

\[
\frac{d\Lambda_t}{d\tau} = c^i \left( M_t^i \right)^{-1} \Omega^i E [zz'] \left[ \Omega_x^i T \left( \Lambda_{t-1}^H, \Lambda_{t-1}^F \right) - \Lambda_{t-1}^i \Omega_x^i \right] ',
\]

\[
\frac{dM_t^i}{d\tau} = c^i \left[ \Omega^i E [zz'] - M_t^i \right].
\]

From Equation (42), it is evident that \( M^i \to \Omega^i E [zz'] \), which implies that Equation (41) reduces to Equation (26).